Section 8-4: Solve Systems by Adding, Subtracting, and Multiplying

By the end of this lesson, you should be able to answer:

- How do you solve systems of equations by adding or subtracting?
- How do you solve systems of equations by adding and multiplying?

Where you might see this in the real world:

- Landscaping, construction, sports

Sometimes, using substitution is not necessarily the best way to solve systems. Today, we will look at adding and subtracting equations in order to eliminate one variable to allow us to find the other. Once we find one variable, we can then substitute into an original equation to find the second variable, then check it with the other equation.

Here are steps used to solve a system by combining equations:

1. Choose a variable to eliminate.
2. If the coefficients of the variables in each equation are opposite already, you can add the two equations together. If the coefficients are not already opposites, multiply each equation to make the coefficients opposite, then add the equations together.
3. Solve the new equation for the variable that is left.
4. Plug the value for this variable back into one of the original equations and solve for the other variable.
5. Check the answer in the other equation and rewrite the answer as an ordered pair.

Example 1: Solve by combining the equations

a. $\begin{cases} -2x + 3y = 2 \\ 2x + 7y = 18 \end{cases}$

b. $\begin{cases} 10x + 4y = 14 \\ x + 4y = 5 \end{cases}$

Notice that in Example 1a you only had to add the two equations together since the coefficients for x were opposites. In Example 1b, you have to multiply one of the equations. You could either multiply either the first or second equation by -1 to eliminate y, or you could multiply the second equation by -10 to eliminate x.

Example 2: Solve by combining the equations

$$\begin{cases} 7x + 2y = 5 \\ 2x + 3y = 16 \end{cases}$$

In Example 2, we need to multiply both equations to make coefficients opposite. If we want to eliminate x, we need to multiply the first equation by -2 and the second one by 7. To eliminate y, we need to multiply the first equation by 3 and the second by -2.

Example 3: Solve the system by combining the equations

$$\begin{cases} 5x + 2y = 7 \\ 4x + 2y = -3 \end{cases}$$

Problem Set:

"A verbal contract isn't worth the paper it's written on." - Samuel Goldwyn